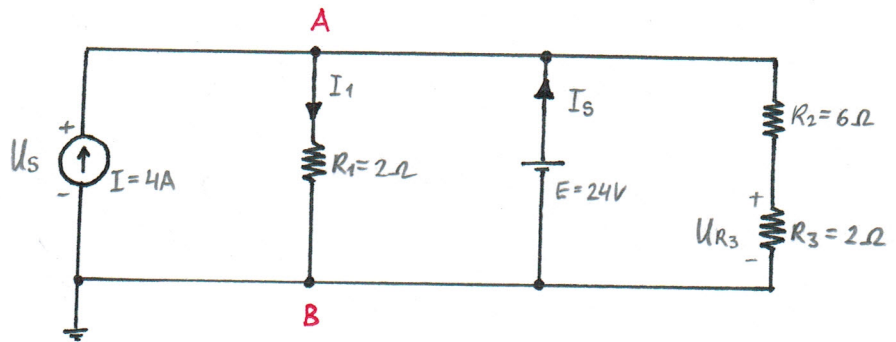


עבודת הבנה - תורת החשמל - שיטת פיתרון

* מאחר E הינו מקור מתח
אידאלי, המתח בין שני
הנודים (U_{AB}) , שווה לערכו
של E

$$U_{AB} = E = 24V$$



$$\textcircled{1} U_I = U_{AB} = 24V$$

$$\textcircled{2} U_{R1} = U_{AB} = 24V$$

$$I_{R1} = \frac{U_{AB}}{R_1} = \frac{24}{2} = 12A$$

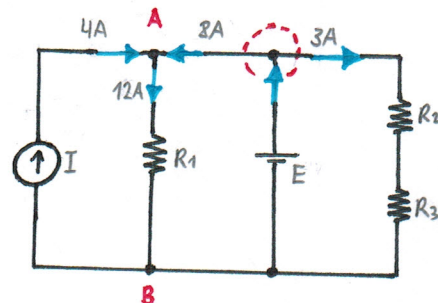
$$\textcircled{2} U_{R3} = E \cdot \frac{R_3}{R_2 + R_3} = 24 \cdot \frac{2}{6+2}$$

$$U_{R3} = 6V$$

$$\textcircled{3} I_{R2,3} = \frac{U_{AB}}{R_2 + R_3} = \frac{24}{6+3} = 3A$$

עם השלמת הפרטים במעגל מתקבל:

$$I(E) = 8 + 3 = 11A$$



$$U_{AB} = \frac{\sum I_{sc}}{\sum G} = \frac{I}{R_1^{-1} + R_2^{-1} + (R_3 + R_4)^{-1}}$$

$$U_{AB} = \frac{0.6}{6^{-1} + 24^{-1} + (16+8)^{-1}}$$

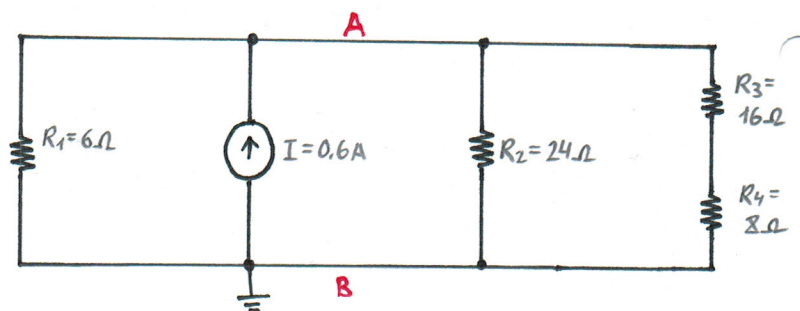
$$U_{AB} = 2.4V$$

$$I_2 = \frac{U_{AB}}{R_2} = \frac{2.4}{24} = 0.1A$$

חוק מתלק המתח עם KVL:

$$U_{R3} = U_{AB} \frac{R_3}{R_3 + R_4} = 2.4 \frac{16}{16+8}$$

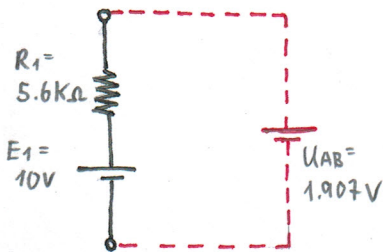
$$U_{R3} = 1.6V$$



$$U_{AB} = \frac{\sum I_{sc}}{\sum G} = \frac{\frac{E_1}{R_1} - \frac{E_2}{R_3} + \frac{E_2}{R_2}}{R_1^{-1} + R_3^{-1} + R_2^{-1}}$$

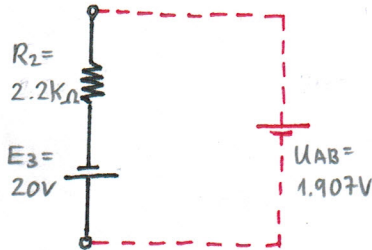
$$U_{AB} = \frac{\frac{10}{5600} - \frac{20}{2200} + \frac{30}{3300}}{\frac{1}{5600} + \frac{1}{2200} + \frac{1}{3300}} = \frac{330}{173}$$

$$U_{AB} = 1.907V$$



$$I_{R1} = \frac{E_1 - U_{AB}}{R_1} = \frac{10 - 1.907}{5600} = \frac{1}{692}$$

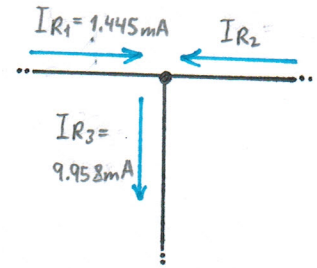
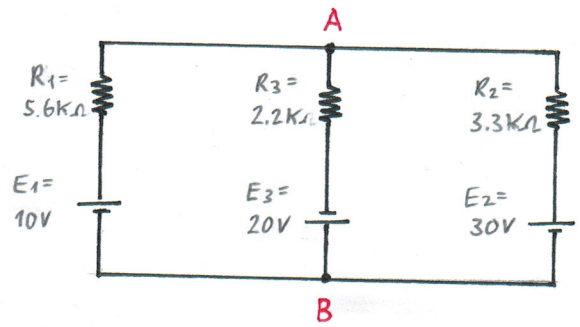
$$I_{R1} = 1.445mA$$



$$I_{R3} = \frac{E_3 + U_{AB}}{R_2} = \frac{20 + 1.907}{2200}$$

$$I_{R3} = 9.958mA$$

$$\therefore \sqrt{N} \ddot{x} - I \sqrt{2} \omega - (2)$$



: KCL @ A - $\ddot{x}$

$$I_{R3} = I_{R1} + I_{R2}$$

$$9.958 = 1.445 + I_{R2}$$

$$I_{R2} = 8.513mA$$

החזקות
mA

חיסוב התנגדויות:

$$R_{11} = R_1 + R_3 = 5.6 + 2.2 = 7.8k\Omega$$

$$R_{12} = R_{21} = R_3 = 2.2k\Omega$$

$$R_{22} = R_2 + R_3 = 2.2 + 3.3 = 5.5k\Omega$$

חיסוב "מתחים":

$$E'_1 = E_1 + E_2 = 10 + 20 = 30V$$

$$E'_2 = -(E_3 + E_2) = -(20 + 30) = -50V$$

$$\begin{cases} +R_{11}I'_1 - R_{12}I'_2 = E'_1 \\ -R_{21}I'_1 + R_{22}I'_2 = E'_2 \end{cases}$$



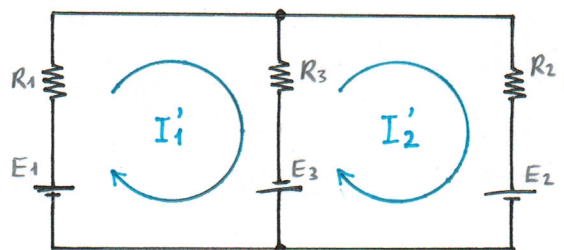
הצבת החילוקים:

$$\begin{cases} +7800I'_1 - 2200I'_2 = 30 \\ 2200I'_1 + 5500I'_2 = (-50) \end{cases}$$

$$I'_1 = \frac{1}{692} = 1.445mA$$

$$I'_2 = \left(-\frac{81}{9515}\right) = (-8.513)mA$$

$$\therefore \sqrt{N} \ddot{x} - I \sqrt{2} \omega - (2)$$



$$I_{R1} = I'_1 = 1.445mA$$

- up

$$I_{R2} = I'_2 = 8.513mA$$

- down

$$I_{R3} = I'_1 - I'_2 = [1.445 - (-8.513)] \times 10^{-3}$$

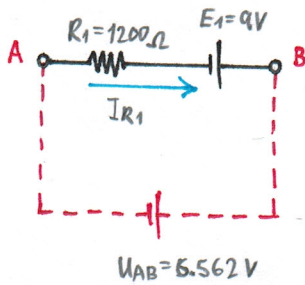
$$I_{R3} = 9.958mA$$

- up

$$U_{AB} = \frac{\sum I_{sc}}{\sum G} = \frac{\frac{-E_1}{R_1} + \frac{0}{R_2} + \frac{E_2}{R_3+R_4}}{R_1^{-1} + R_2^{-1} + (R_3+R_4)^{-1}}$$

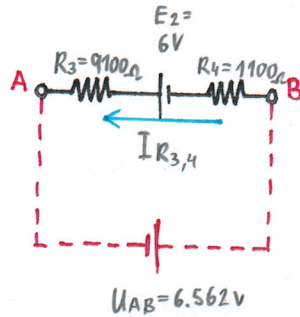
$$U_{AB} = \frac{\frac{-9}{1200} + 0 + \frac{6}{9100+1100}}{1200^{-1} + 8200^{-1} + (9100+1100)^{-1}} = \left(-\frac{5781}{881} \right)$$

$$U_{AB} = (-6.562) V$$



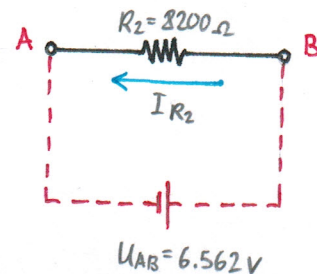
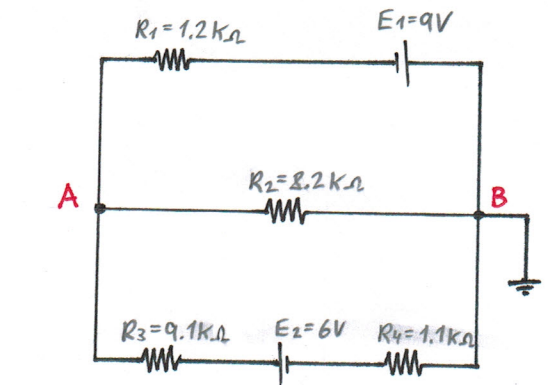
$$I_{R1} = \frac{E_1 - U_{AB}}{R_1} = \frac{9 - 6.562}{1200}$$

$$I_{R1} = 2.032 mA$$



$$I_{R3} = I_{R4} = \frac{E_2 + U_{AB}}{R_3 + R_4} = \frac{6 + 6.562}{9100 + 1100}$$

$$I_{R3} = I_{R4} = 1.231 mA$$



$$I_{R2} = \frac{U_{AB}}{R_1} = \frac{6.562}{8200}$$

$$I_{R2} = 0.8 mA$$

חישוב ההתנגדויות בחוגים:

$$R_{11} = R_1 + R_2 = 1200 + 8200 = 9400 \Omega$$

$$R_{12} = R_{21} = R_2 = 8200 \Omega$$

$$R_{22} = R_2 + R_3 + R_4 = 8200 + 9100 + 1100 = 18400 \Omega$$

חישוב המתחים:

$$E'_1 = 9V$$

$$E'_2 = 6V$$

$$\begin{pmatrix} +R_{11} & -R_{12} \\ -R_{21} & +R_{22} \end{pmatrix} \begin{pmatrix} I'_1 \\ I'_2 \end{pmatrix} = \begin{pmatrix} E'_1 \\ E'_2 \end{pmatrix}$$

$$\begin{pmatrix} +9400 & -8200 \\ -8200 & +18400 \end{pmatrix} \begin{pmatrix} I'_1 \\ I'_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$$

הצבת החישובים במערכת:

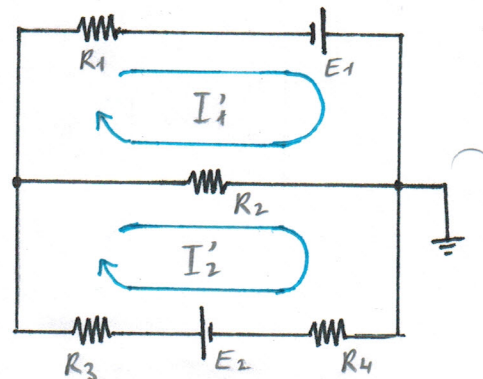
$$I'_1 = \frac{179}{88100} = 2.032 mA$$

$$I_{R1} = I'_1 = 2.032 mA$$

$$I'_2 = \frac{217}{176200} = 1.231 mA$$

$$I_{R3} = I_{R4} = I'_2 = 1.231 mA$$

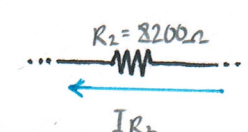
2



$$I_{R2} = I'_1 - I'_2$$

$$I_{R2} = (2.032 - 1.231) \times 10^{-3}$$

$$I_{R2} = 0.8 mA \quad \text{left}$$



③ מצא I - שיטת מילמן: (מקרה פרטי של "מתח 3 מתחים" - עם 2 מתחים)

$$V_1 = \frac{\sum I_{SC}}{\sum G} = \frac{\frac{-E_1}{R_1} + \frac{-E_2}{R_3} + \frac{E_2 - E_4}{R_2}}{R_1^{-1} + R_3^{-1} + R_2^{-1}}$$

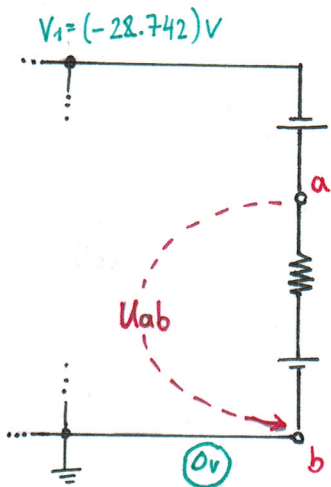
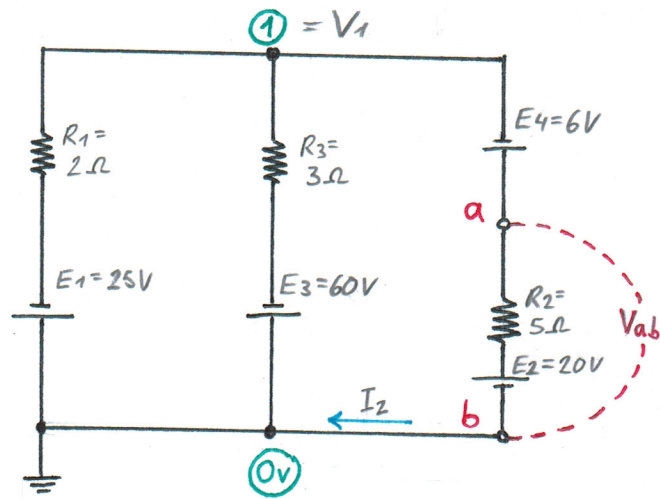
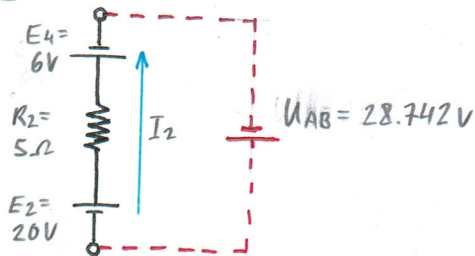
$$V_1 = \frac{\frac{-25}{2} + \frac{-60}{3} + \frac{20-6}{5}}{2^{-1} + 3^{-1} + 5^{-1}} = \frac{-891}{31}$$

$$V_1 = (-28.742) V$$

$$I_2 = \frac{+V_1 + E_2 - E_4}{R_2} = \frac{+28.742 + 20 - 6}{5} = \frac{265}{31}$$

$$I_2 = 8.548 A$$

*כיוון הזרם מתחשב
השרטוט המקורי



אם לא מצא את המתח U_{ab} - נבצר "I" -
לאסוף המתחים מקובקב A עד לקובקב B
נבד כיון השלסן:

$$U_{ab} = +E_4 + V_1$$

$$U_{ab} = +6 + (-28.742)$$

$$U_{ab} = (-22.742) V$$

③ מציאת "זרמי חושים":

מערכת המשוואות:

$$\begin{pmatrix} +R_{11} & -R_{21} \\ -R_{21} & +R_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1' \\ I_2' \end{pmatrix} = \begin{pmatrix} E_1' \\ E_2' \end{pmatrix}$$

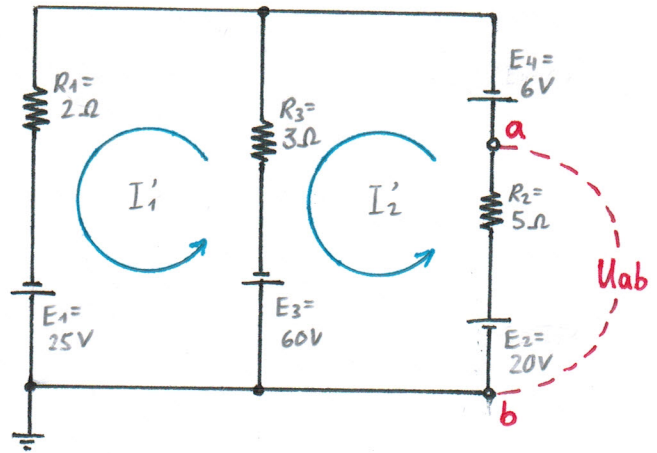
חישוב התנגדותות:

$$R_{11} = R_1 + R_2 = 2 + 3 = \boxed{5\Omega}$$

$$R_{12} = R_{21} = R_3 = \boxed{3\Omega}$$

$$R_{22} = R_2 + R_3 = 5 + 3 = \boxed{8\Omega}$$

$$\begin{aligned} E_1' &= E_1 - E_3 \\ &= 25 - 60 = \boxed{(-35)V} \\ E_2' &= E_3 + E_2 - E_4 \\ &= 60 + 20 - 6 = \boxed{74V} \end{aligned}$$



הצבת המשוואות במערכת:

$$\begin{pmatrix} +5 & -3 \\ -3 & +8 \end{pmatrix} \cdot \begin{pmatrix} I_1' \\ I_2' \end{pmatrix} = \begin{pmatrix} -35 \\ 74 \end{pmatrix}$$

$$I_1' = \left(-\frac{58}{31} \right) = \boxed{(-1.871)A}$$

$$I_2' = \frac{265}{31} = \boxed{8.548A}$$

$$\rightarrow \boxed{I_2 = I_2' = 8.548A}$$

* הכיוון ההפוך מהמסומן בשאלה

חישוב U_{ab} :

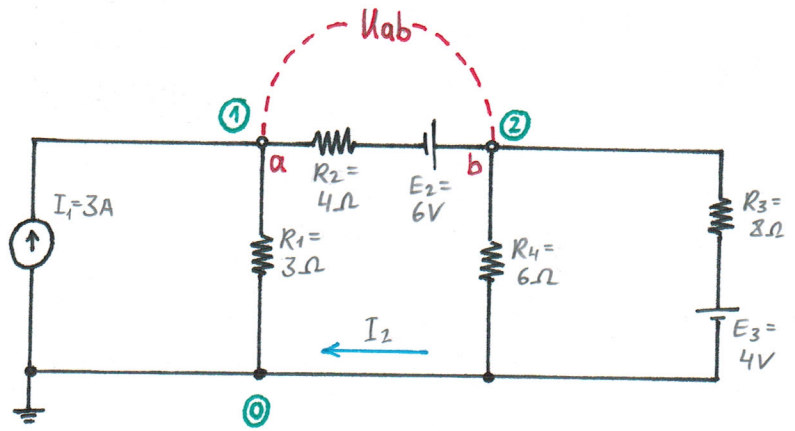
$$U_{R2} = I_2 \cdot R_2 = 8.548 \cdot 5$$

$$\boxed{U_{R2} = 42.741V}$$

$$U_{ab} = -U_{R2} + E_2 = -42.741 + 20 = \frac{-705}{31}$$

$$\boxed{U_{ab} = (-22.742)V}$$

$$\begin{pmatrix} +G_{11} & -G_{12} \\ -G_{21} & +G_{22} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_{SC1} \\ I_{SC2} \end{pmatrix}$$



$$G_{11} = R_1^{-1} + R_2^{-1} = 3^{-1} + 4^{-1} = \boxed{\frac{7}{12} S}$$

$$G_{12} = G_{21} = R_2^{-1} = \boxed{0.25 S}$$

$$G_{22} = R_2^{-1} + R_4^{-1} + R_3^{-1} = 4^{-1} + 6^{-1} + 8^{-1} = \boxed{\frac{13}{24} S}$$

$$I_{SC1} = I_1 - \frac{E_2}{R_2} + \frac{0}{R_1} = 3 - \frac{6}{4} + 0 = \boxed{1.5 A}$$

$$I_{SC2} = \frac{E_2}{R_2} + \frac{0}{R_4} + \frac{E_3}{R_3} = \frac{6}{4} + 0 + \frac{4}{8} = \boxed{2 A}$$

$$\begin{pmatrix} +\frac{7}{12} & -0.25 \\ -0.25 & \frac{13}{24} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 2 \end{pmatrix}$$

$$V_1 = \frac{378}{73} = 5.178 V$$

$$V_a = V_1$$

$$V_2 = \frac{444}{73} = 6.082 V$$

$$V_b = V_2$$

$$U_{ab} = V_a - V_b$$

$$U_{ab} = 5.178 - 6.083$$

$$U_{ab} = -\frac{66}{73}$$

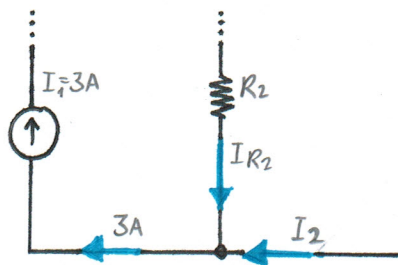
$$U_{ab} = \boxed{(-0.904) V}$$

נשאלת 3 נקודות: I_2 ו- I_{R1}

$$U_{R1} = V_a = 5.178 V$$

$$I_{R1} = \frac{U_{R1}}{R_1} = \frac{5.178}{3} = \frac{126}{73}$$

$$I_{R1} = \boxed{1.726 A}$$



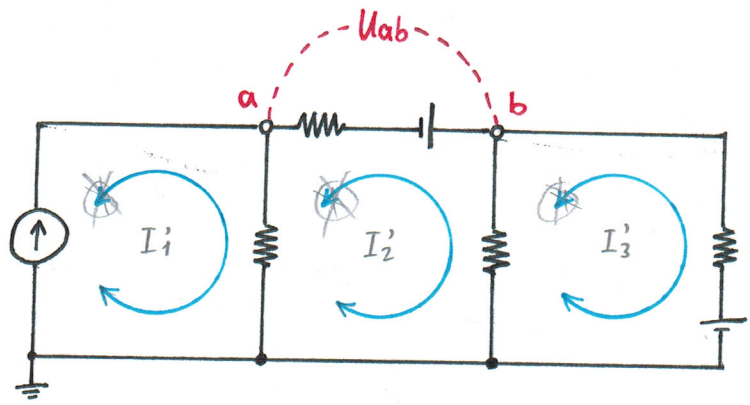
$$I_2 = 3 - I_{R2} = 3 - 1.726 = \frac{93}{77}$$

$$I_2 = \boxed{1.274 A}$$

3 מעגל II - בשיטת "צורה חוזרים":

* מקור הזרם האידיאלי בחישוב I_1' - "נפיל" את המשוואה הראשונה.

$$\begin{pmatrix} +R_{11} & -R_{12} & -R_{13} \\ -R_{21} & +R_{22} & -R_{23} \\ -R_{31} & -R_{32} & +R_{33} \end{pmatrix} \cdot \begin{pmatrix} I_1' \\ I_2' \\ I_3' \end{pmatrix} = \begin{pmatrix} E_1' \\ E_2' \\ E_3' \end{pmatrix}$$



חישוב ההתנגדויות:

חישוב המתחים:

$$R_{12} = R_{21} = R_1 = 3\Omega$$

$$E_2' = E_2$$

$$R_{22} = R_1 + R_2 + R_4$$

$$E_2' = 6V$$

$$4 + 3 + 6 = 13\Omega$$

$$E_3' = (-E_3)$$

$$R_{23} = R_{32} = R_4 = 6\Omega$$

$$E_3' = (-4)$$

$$R_{31} = R_{13} = 0$$

$$R_{33} = R_4 + R_3 = 6 + 8 = 14\Omega$$

התחבת משוואה: מאחר וקיים מקור זרם אידיאלי I_1

בחישוב I_1' , נאלץ "לנפח" את

המשוואה הראשונה:

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 13 & -6 \\ 0 & -6 & 14 \end{pmatrix} \cdot \begin{pmatrix} I_1' \\ I_2' \\ I_3' \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -4 \end{pmatrix}$$

$$I_1' = 3A$$

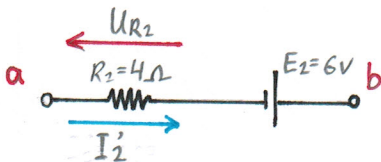
$$I_2' = \frac{93}{73} = 1.274A$$

$$I_2 = I_2' = 1.274A$$

$$I_3' = \frac{19}{73} = 0.26A$$

כיוון: כחיסור בפרט.

חישוב U_{ab} :



$$U_{R2} = I_2' \cdot R_2 = 1.274 \cdot 4$$

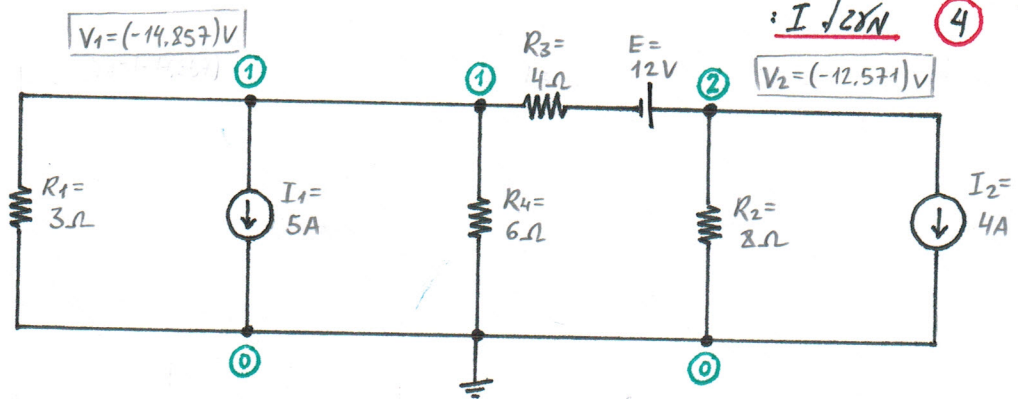
$$U_{ab} = U_{R2} - E_2 = 5.096 - 6 = \frac{66}{73}$$

$$U_{R2} = 5.096V$$

$$U_{ab} = 0.904V$$

מערכת הכתובות:

4 $I \sqrt{2} N$



חישוב המוליכויות:

חישוב זרמי הקצרי:

$$G_{11} = R_4^{-1} + R_1^{-1} + R_3^{-1} = 6^{-1} + 3^{-1} + 4^{-1} = 0.75 \text{ S}$$

$$I_{SC1} = \frac{-E}{R_2} + I_1 = \frac{-12}{4} - 5$$

$$I_{SC1} = (-8) \text{ A}$$

$$G_{12} = G_{21} = R_3^{-1} = 0.25 \text{ S}$$

$$I_{SC2} = \frac{E}{R_3} - I_2 = \frac{12}{4} - 4$$

$$I_{SC2} = (-1) \text{ A}$$

$$G_{22} = R_2^{-1} + R_3^{-1} = 8^{-1} + 4^{-1} = 0.375 \text{ S}$$

הצבת החישובים במערכת:

$$\begin{pmatrix} +0.75 & -0.25 \\ -0.25 & +0.375 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$$

$$\begin{aligned} V_1 &= \left(-\frac{104}{7} \right) = (-14.857) \text{ V} \\ V_2 &= \left(-\frac{88}{7} \right) = (-12.571) \text{ V} \end{aligned}$$

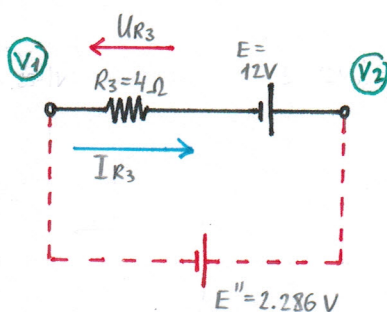
* נוסח בלתי נכון:

① = V_1

② = V_2

$$U_{R1} = U_{R4} = |V_1| = 14.857 \text{ V} \text{ down}$$

$$U_{R2} = V_2 = |V_2| = 12.571 \text{ V} \text{ down}$$



$$E'' = V_2 - V_1$$

$$E'' = (-12.571) - (-14.857)$$

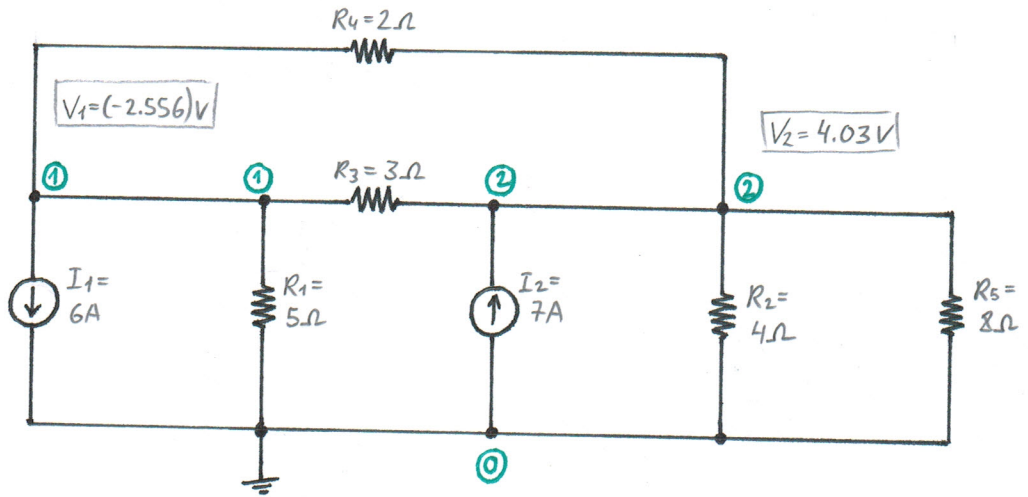
$$E'' = \frac{16}{7} = 2.286 \text{ V}$$

$$U_{R3} = E - E'' = 12 - 2.286$$

$$U_{R3} = 9.714 \text{ V} \text{ left}$$

מערכת המשוואות:

$$\begin{pmatrix} +G_{11} & -G_{12} \\ -G_{21} & +G_{22} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_{sc1} \\ I_{sc2} \end{pmatrix}$$



חישוב ההתנגדויות:

$$G_{11} = R_3^{-1} + R_1^{-1} + R_4^{-1} = 3^{-1} + 5^{-1} + 2^{-1} = \frac{31}{30} S$$

$$G_{12} = G_{21} = R_3^{-1} + R_4^{-1} = 3^{-1} + 2^{-1} = \frac{5}{6} S$$

$$G_{22} = R_3^{-1} + R_2^{-1} + R_5^{-1} + R_4^{-1} = 3^{-1} + 4^{-1} + 8^{-1} + 2^{-1} = \frac{29}{24} S$$

חישוב זרמי הקצרי:

$$I_{sc1} = (-I_1)$$

$$I_{sc1} = (-6) A$$

$$I_{sc2} = I_2$$

$$I_{sc2} = 7$$

הצבת החישובים במערכת:

$$\begin{pmatrix} +\frac{31}{30} & -\frac{5}{6} \\ -\frac{5}{6} & +\frac{29}{24} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}$$

$$V_1 = \frac{-340}{133} = (-2.556) V$$

$$V_2 = \frac{536}{133} = 4.03 V$$

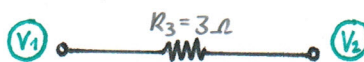
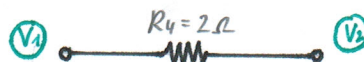
סימן במכשיר *

$$① = V_1$$

$$② = V_2$$

$$U_{R1} = V_1 = 2.556 V \text{ down}$$

$$U_{R2} = U_{R5} = V_2 = 4.03 V \text{ up}$$



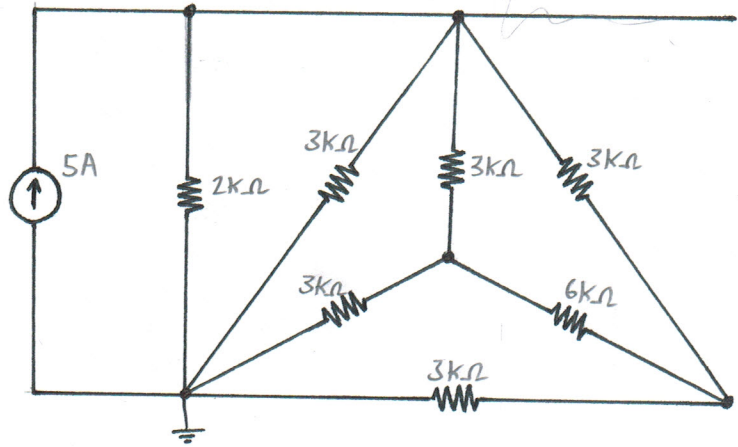
$$U_{R3} = U_{R4} = V_2 - V_1$$

$$U_{R3} = U_{R4} = 4.03 - (-2.556) = \frac{876}{133}$$

$$U_{R3} = U_{R4} = 6.586 V \text{ right}$$

מערכת המשוואות:

$$\begin{pmatrix} +G_{11} & -G_{12} & -G_{13} \\ -G_{21} & +G_{22} & -G_{23} \\ -G_{31} & -G_{32} & +G_{33} \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} I_{sc1} \\ I_{sc2} \\ I_{sc3} \end{pmatrix}$$



חישוב המוליכויות:

$$G_{11} = 2000^{-1} + (3000^{-1}) \cdot 3 = \frac{3}{2000} S$$

$$G_{12} = G_{21} = \frac{1}{3000} S$$

$$G_{13} = G_{31} = \frac{1}{3000} S$$

$$G_{22} = 6000^{-1} + (3000^{-1}) \cdot 2 = \frac{1}{1200} S$$

$$G_{23} = G_{32} = \frac{1}{6000} S$$

$$G_{33} = 6000^{-1} + (3000^{-1}) \cdot 2 = \frac{1}{1200} S$$

חישוב זרמי הקצרי:

$$I_{sc1} = 5A$$

$$I_{sc2} = 0$$

$$I_{sc3} = 0$$

הצבת החישובים במערכת:

$$\begin{pmatrix} +\frac{3}{2000} & -\frac{1}{3000} & -\frac{1}{3000} \\ -\frac{1}{3000} & +\frac{1}{1200} & -\frac{1}{6000} \\ -\frac{1}{3000} & -\frac{1}{6000} & +\frac{1}{1200} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

||

$$V_1 = \frac{30000}{7} = 4285.714 V$$

||

$$I = \frac{V_1}{2000} = \frac{4285.714}{2000}$$

$$I = 2.143 A$$

*כיוון הזרם
השואל המקורי